Int. J. Heat Mass Transfer. Vol. 30, No. 12, pp. 2651–2656, 1987
Printed in Great Britain

0017-9310/87 $3.00+0.00
Pergamon Journals Ltd.

Natural convection with combined heat and mass transfer buoyancy effects in non-homogeneous porous medium

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(*Received* 21 *July* 1986 *and in final form* 25 *November* 1986)

Abstract—The effect of nonhomogeneity of the medium is investigated in the case of natural convection in a vertical porous layer. The flows are driven by conditions of uniform heat and mass fluxes imposed along the two vertical side walls of the porous layer. It is found that the effect of nonhomogeneity is significant for sharper permeability variations and large Rayleigh numbers, Lewis numbers and absolute buoyancy ratios. Nusselt and Sherwood number estimates in such cases are found to be quite different from that of the homogeneous medium.

INTRODUCTION

THIS PAPER analyses the effect of nonhomogeneity on natural convection with combined heat and mass transfer buoyancy effects in a non-homogeneous porous medium saturated with fluid. The flows are driven by conditions of uniform heat and mass fluxes imposed along the two vertical side walls of the porous layer. The engineering applications of the problem are important; e.g. the migration of moisture through air contained in fibrous insulations and grain storage installations and the dispersion of chemical contaminants through water saturated soil. A study of the problem with a homogeneous medium was carried out by Trevisan and Bejan [1].

Most of the problems carried out in the area of heat and mass transfer in porous media assume homogeneity of the medium. In real life, however, most porous insulation systems are nonhomogeneous partly due to fabrication procedures and partly due to installation by human hand. Due to the difference in types of soils at various depths [2] ground water pollution can also be considered as dispersion in a non-homogeneous porous medium. A layered porous medium has been considered [3] in the problem of natural convection through a porous medium heated from the side. But such a model may not always be realistic as in many cases discrete layers of different permeabilities do not exist. Variation of permeability in such cases is a continuous one.

The effect of variable permeability with a continuously varying profile was studied by Ribando and Torrence [4] in the context of natural convection of a fluid in a porous medium heated from below. Facas and Farouk [5] studied the effect of variable permeability in the case of natural convection in a porous medium between two concentric cylinders. Another paper [6] studied the influence of variable permeability on combined free and forced convection about inclined surfaces in porous media. The objective of this paper is to study the effect of nonhomogeneity on natural convection with combined heat and mass transfer by assuming continuously varying profiles for permeability.

According to the Kozeny–Carman equation [7] the permeability is given by porosity and particle size of the medium, assuming a constant shape factor. However, dependence on particle size is more significant than dependence on porosity [2]. Moreover, in many cases variations in porosity are usually of limited extent [2]. Therefore, in this study we assume that the nonhomogeneity of the porous medium results from particle size variation. The porosity is assumed constant. Consequently thermal conductivity and thermal diffusivity of the porous medium saturated with fluid are assumed constant here [4]. Also we take mass diffusivity D to be constant as done by Trevisan and Bejan [1].

FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

Consider the two-dimensional rectangular porous medium as shown in Fig. 1. The isotropic non-homogeneous medium is assumed fluid saturated. The vertical walls are subject to uniform fluxes of heat and mass

$$q'' = -\bar{k}\frac{\partial T}{\partial x}\bigg|_{x=0,l} \tag{1}$$

$$j'' = -D\frac{\partial c}{\partial x}\bigg|_{x=0,l} \tag{2}$$

Equations (1) and (2) mean heating and mass influx

NOMENCLATURE

c	concentration
c_0	concentration measured at the centre of the porous layer $x = l/2, y = 0$
ΔC	side to side concentration difference
C	non-dimensional concentration, $(c-c_0)/(j''h/D)$
D	mass diffusivity
g	gravitational acceleration
h	height of porous layer
j''	lateral mass flux
k	permeability
k_0	reference permeability
$\bar{k}$	thermal conductivity
K	defined by the relation $k = k_0K(X, Y)$
l	horizontal thickness of porous layer
Le	Lewis number
N	buoyancy ratio
Nu	Nusselt number
p	pressure
q''	lateral heat flux
Ra	Rayleigh number
Sh	Sherwood number
T	temperature
T_0	temperature measured at the centre of the porous layer $x = l/2, y = 0$
T'	non-dimensional temperature, $(T-T_0)/(q''h/k)$
ΔT	side to side temperature difference
u, v	velocity components
x, y	Cartesian coordinates
X	horizontal position, x/h
Y	lateral position, $(y+h/2)/h$.

Greek symbols

α	thermal diffusivity
$\alpha_1, \alpha_2, \alpha_3$	constants as defined in equation (8)
β	thermal expansion coefficient
β_c	concentration expansion coefficient
ν	kinematic viscosity
ψ	stream function such that $u = \partial\psi/\partial y, v = -\partial\psi/\partial x$
ψ'	non-dimensional stream function, $\psi/(\alpha Ra)$.

at $x = 0$ and cooling combined with mass efflux at $x = l$. The symbols are defined in the nomenclature.

The non-dimensional governing equations assuming a Boussinesq incompressible fluid are [1]

$$\frac{\partial}{\partial X}\left(\frac{1}{K(X,Y)}\frac{\partial \psi'}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{1}{K(X,Y)}\frac{\partial \psi'}{\partial X}\right) = -\frac{\partial T'}{\partial X}-N\frac{\partial C}{\partial X} \quad (3)$$

$$Ra\left(\frac{\partial \psi'}{\partial Y}\frac{\partial T'}{\partial X}-\frac{\partial \psi'}{\partial X}\frac{\partial T'}{\partial Y}\right)=\frac{\partial^2 T'}{\partial X^2}+\frac{\partial^2 T'}{\partial Y^2} \quad (4)$$

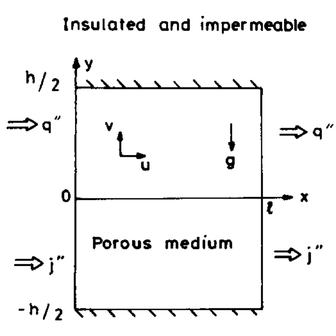

FIG. 1. Schematic diagram of a two-dimensional porous layer subjected to uniform heat and mass fluxes in the horizontal direction.

$$Le\,Ra\left(\frac{\partial \psi'}{\partial Y}\frac{\partial C}{\partial X}-\frac{\partial \psi'}{\partial X}\frac{\partial C}{\partial Y}\right)=\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2} \quad (5)$$

with boundary conditions

$$\psi' = 0, \quad \frac{\partial T'}{\partial X}=\frac{\partial C}{\partial X}=-1 \quad \text{at } X = 0, l/h$$
$$\psi' = 0, \quad \frac{\partial T'}{\partial Y}=\frac{\partial C}{\partial Y}=0 \quad \text{at } Y = 0, 1 \quad (6)$$

where

$$Ra = \frac{k_0 g\beta h^2 q''}{\alpha\nu\bar{k}}$$

$$Le = \frac{\alpha}{D}$$

$$N = \frac{\beta_c}{\beta}\frac{j''}{q''}\frac{\bar{k}}{D}$$

and

$$k = k_0K(X, Y).$$

T_0 and C_0 are temperature and concentration measured at the centre of the porous medium $x = l/2$, $y = 0$.

Finite-difference equations are derived for equations (3)–(5) by integration over finite area elements as outlined by Gosman *et al.* [8]. The simultaneous algebraic equations were solved using an iterative point by point method and relaxation parameters. The Nusselt number and Sherwood number defined

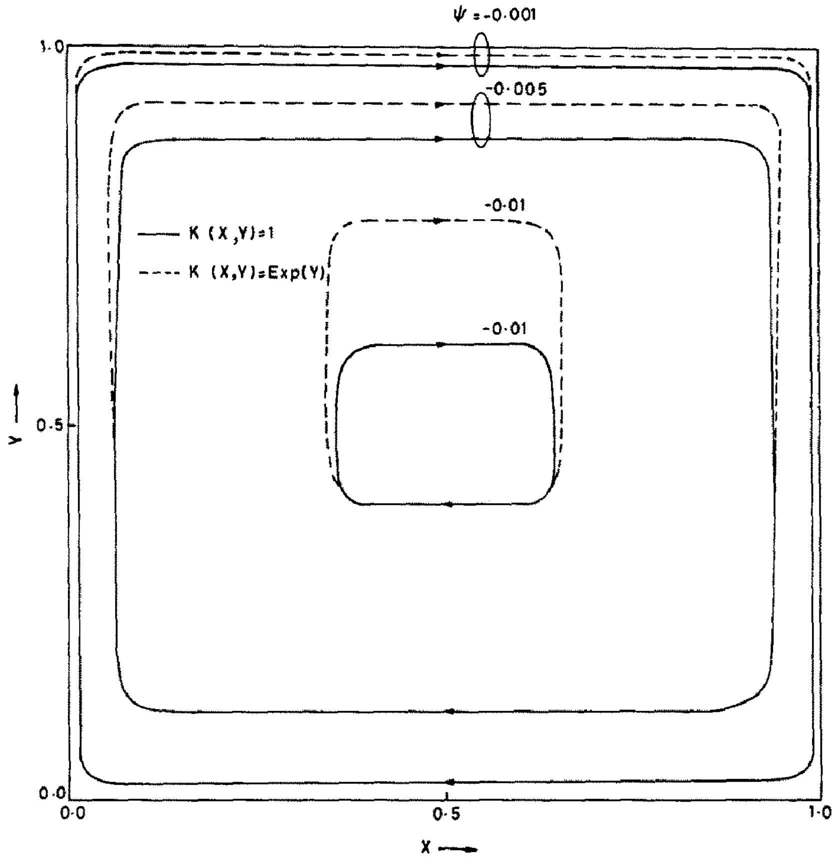

FIG. 2. Patterns of streamlines for homogeneous and non-homogeneous cases with $Ra = 400$, $Le = 1$ and $N = 1$.

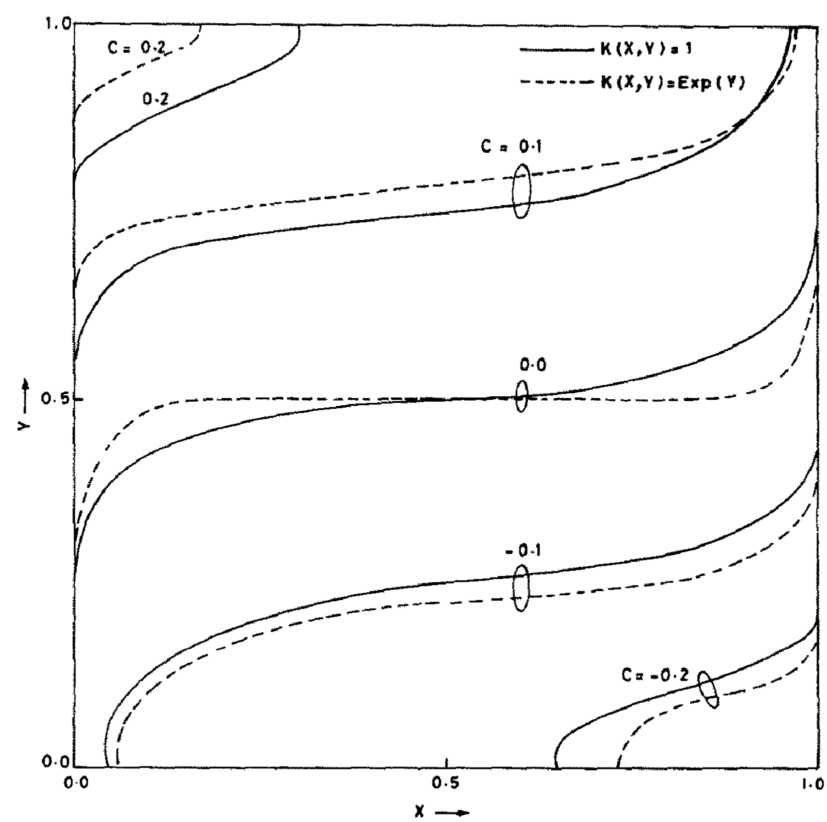

FIG. 3. Constant concentration lines for homogeneous and non-homogeneous cases with $Ra = 400$, $Le = 1$ and $N = 1$.

below were calculated

$$Nu = \frac{l}{h}\left(\int_0^1 \Delta T' \, \mathrm{d}Y\right)^{-1}$$
$$Sh = \frac{l}{h}\left(\int_0^1 \Delta C \, \mathrm{d}Y\right)^{-1}. \quad (7)$$

A non-uniform grid system with maximum grid points near the boundaries was adopted in order to get accurate and fast convergent solutions. *Ra*, *Le* and *N* were given a realistic range of values. The aspect ratio l/h was fixed as one. Different permeability profiles were considred for the porous medium. The motive behind the choices was to demonstrate the effect of non-homogeneity. In fact permeability variation differs in different porous media. However, as a first step in this direction we adopted parameterized profiles as

$$k = k_0(1+\alpha_1 X+\alpha_2 Y)^{\alpha_3}$$
$$k = k_0 \exp(\alpha_1 X+\alpha_2 Y). \quad (8)$$

NUMERICAL RESULTS AND DISCUSSION

The results are presented in the form of graphs and tables. Figure 2 presents the streamlines for homogeneous and horizontally stratified cases and Fig. 3 presents the corresponding constant concentration lines. We find that streamlines are denser in the regions of high permeability. Vigorous convection through highly permeable areas result in less side to side concentration difference compared to the homogeneous case as evident from Table 1. Table 1 gives Nusselt number, Sherwood number for various permeability profiles. The results show that nonhomogeneity can have a significant effect on *Nu* and *Sh* depending of course on the assumed non-homogeneity profile. Table 2 shows that for large values of *Ra*, a difference in the estimation of *Nu* and *Sh* compared to the homogeneous case is more significant. For a large Lewis number and large absolute values of buoyancy ratio the assumption of nonhomogeneity of the medium has an even greater effect on *Nu* and *Sh*. This can be inferred from Tables 3 and 4.

CONCLUSION

The effect of nonhomogeneity of the porous medium is studied for different permeability profiles, with large ranges of values for Rayleigh number, Lewis number and buoyancy ratio. It was found that vigorous convection through high permeability regions make the temperature and concentration in the porous layer quite different from that of the homogeneous case. An assumption of homogeneity in the case of a non-homogeneous medium would result in unsatisfactory estimates of *Nu* and *Sh*.

Table 1. Estimation of *Nu*, *Sh* for different permeability profiles with $Ra = 400$, $Le = 1$ and $N = 1$

Permeability profile $K(X, Y)$	1	$1+Y$	$1/(1+Y)$	$1+X$	$1/(1+X)$	$1+X+Y$	$1/(1+X+Y)$	$\exp(Y)$	$\exp(-Y)$	$\exp(X)$	$\exp(-X)$	$\exp(X+Y)$	$\exp(-X-Y)$
Nu, Sh	5.55	6.51	4.76	6.44	4.75	7.24	4.25	6.82	4.55	6.73	4.47	8.12	3.69
$\Delta T'$, ΔC $(Y = 0)$	0.239	0.231	0.247	0.209	0.272	0.205	0.279	0.230	0.248	0.201	0.286	0.196	0.298
$\Delta T'$, ΔC $(Y = 1)$	0.240	0.193	0.295	0.214	0.272	0.180	0.315	0.175	0.317	0.207	0.284	0.152	0.372
$\Delta T'$, ΔC $(Y = 1/2)$	0.156	0.131	0.182	0.134	0.182	0.119	0.205	0.125	0.190	0.128	0.195	0.106	0.236

Table 2. Numerical results for different values of *Ra* with $Le = 1$ and $N = 0$

	$K(X, Y) = 1$				$K(X, Y) = \exp(Y)$				Difference in *Nu*, *Sh* compared to homogeneous case
Ra	*Nu*, *Sh*	$\Delta T', \Delta C$ $Y = 0$	$\Delta T', \Delta C$ $Y = 1$	$\Delta T', \Delta C$ $Y = 1/2$	*Nu*, *Sh*	$\Delta T', \Delta C$ $Y = 0$	$\Delta T', \Delta C$ $Y = 1$	$\Delta T', \Delta C$ $Y = 1/2$	
50	1.72	0.581	0.580	0.536	2.14	0.533	0.473	0.400	0.42
100	2.30	0.480	0.480	0.373	2.85	0.447	0.381	0.308	0.55
500	4.65	0.295	0.294	0.160	5.54	0.269	0.207	0.158	0.89
1000	6.20	0.220	0.222	0.138	7.57	0.214	0.161	0.112	1.37
5000	14.11	0.127	0.121	0.054	15.70	0.125	0.092	0.051	1.59
10000	16.49	0.100	0.100	0.049	18.36	0.099	0.074	0.048	1.87

Table 3. *Nu*, *Sh* for different values of *Le* with $Ra = 400$ and $N = 0$

	$K(X, Y) = 1$		$K(X, Y) = \exp(Y)$		Difference in *Sh* compared to homogeneous case
Le	*Nu*	*Sh*	*Nu*	*Sh*	
0.1	4.18	1.10	5.11	1.13	0.03
0.5	4.16	1.98	5.10	2.15	0.17
1.0	4.16	4.16	5.10	5.10	0.94
5.0	4.16	12.62	5.10	15.12	2.50
10.0	4.15	16.56	5.08	20.65	4.09

Table 4. *Nu*, *Sh* for different values of *N* with $Ra = 400$ and $Le = 1$

N	$K(X, Y) = 1$ *Nu*, *Sh*	$K(X, Y) = \exp(Y)$ *Nu*, *Sh*	Difference in *Nu*, *Sh* compared to homogeneous case
−5	7.25	8.58	1.33
−1	1.0	1.0	0.00
0	4.16	5.10	0.94
1	5.55	6.82	1.27
5	8.64	10.43	1.79

Acknowledgement—The authors wish to express their grateful thanks to the referee for his valuable comments.

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CONVECTION NATURELLE AVEC EFFETS COMBINES DE CHALEUR ET DE MASSE DANS UN MILIEU POREUX HETEROGENE

Résumé—On étudie l'effet de l'héterogénéité du milieu dans le cas de la convection naturelle dans une couche poreuse verticale. Les écoulements sont pilotés par des conditions de flux de chaleur et de masse uniformes imposées le long des deux parois verticales de la couche poreuse. On trouve que l'effet de l'héterogénéité est sensible pour des variations importantes de perméabilité et de grandes valeurs du nombres de Rayleigh, du nombre de Lewis et du rapport absolu de flottement. Les estimations des nombres de Nusselt et de Sherwood dans de telles conditions sont très différentes de celles obtenues pour un milieu homogène.

WÄRME- UND STOFFTRANSPORT BEI NATÜRLICHER KONVEKTION-AUFTRIEBSEFFEKTE IN EINEM INHOMOGENEN PORÖSEN MEDIUM

Zusammenfassung—In einer senkrechten porösen Schicht wird der Einfluß von Inhomogenitäten bei natürlicher Konvektion untersucht. Die Strömung wird durch einheitliche aufgeprägte Wärme- und Stoffströme entlang der zwei senkrechten Begrenzungswände der porösen Schicht angetrieben. Der Einfluß von Inhomogenitäten ist stärker, wenn sich die Porosität plötzlich stark ändert, und bei großen Rayleigh-Zahlen, großen Lewis-Zahlen und großem absoluten Auftriebsverhältnis. Berechnete Nusselt- und Sherwood-Zahlen weichen in diesen Fällen erheblich von denen im homogenen Medium ab.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ С УЧЕТОМ ВЗАИМОСВЯЗАННЫХ ПРОЦЕССОВ ТЕПЛО- И МАССООБМЕНА В НЕОДНОРОДНОЙ ПОРИСТОЙ СРЕДЕ

Аннотация—Рассматривается влияние неоднородности среды при естественной конвекции в вертикальном пористом слое. Течения формируются однородными потоками тепла и массы вдоль обеих вертикальных и боковых стенок пористого слоя. Найдено, что влияние неоднородности существенно в случаях более резких изменений проницаемости, больших значений чисел Рэлея и Льюиса и абсолютного отношения сил плавучести. Оценки чисел Нуссельта и Шервуда в этих случаях совершенно отличны от значений, полученных для однородной среды.